Answers Home Work 3

3.1

Because of symmetric condition, half portion is considered in the energy calculation.

(1) Displacement diagram of symmetrical mechanism is shown in Fig.1-1.

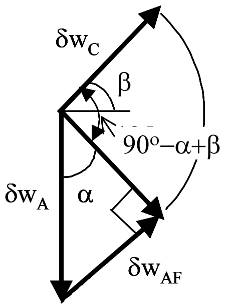

Fig.1-1

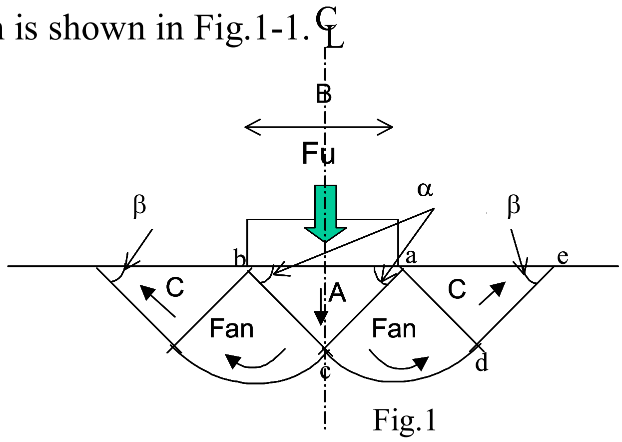

Fig.1

Table 1-1

slip plane	shear stress c_u	length L	displacement δw	$\delta W = c_u L \delta w$
ac	c_u	$B/(2\cos\alpha)$	$\delta w_a \sin\alpha$	$Bc_u\delta w_a \sin\alpha/(2\cos\alpha)$
Fan	c_u	$R=B/(2\cos\alpha)$ $\delta\theta=1/2\pi-\alpha+\beta$	$\delta w_a \cos\alpha$	$Bc_u\delta w_a(1/2\pi-\alpha+\beta)$
ad	-	-	0	0
ed	c_u	$B\cos\beta/(2\cos\alpha\sin\beta)$	$\delta w_a \cos\alpha$	$Bc_u\delta w_a \cos\beta/(2\sin\beta)$

(2) From Table 1-1, total internal dissipation δW for the whole portion is given by eq(1)

$$\delta W = B\delta w_a c_u\left(\frac{\sin\alpha}{\cos\alpha}+\pi-2\alpha+2\beta+\frac{\cos\beta}{\sin\beta}\right) \qquad (1)$$

(3) By equating the external energy $\delta E(=F_u\delta w_\alpha)$ to internal dissipation (eq.(1), the following bearing capacity equation is obtained.

$$F_u = Bc_u\left(\frac{\sin\alpha}{\cos\alpha}+\pi-2\alpha+2\beta+\frac{\cos\beta}{\sin\beta}\right) \qquad (2)$$

(4) Minimum upper bound of bearing capacity of the mechanism is given by the following condition.

$$\frac{dF_u}{d\alpha}=0,\quad \frac{dF_u}{d\beta}=0 \quad (3) \quad \Rightarrow \quad \frac{1}{\cos^2\alpha}-2=0,\quad 2-\frac{1}{\sin^2\beta}=0 \quad \Rightarrow \quad \alpha=\beta=\pi/4 \qquad (4)$$

inserting eq. (4) into eq.(2) Minimum F_u is $\quad F_u = Bc_u(\pi+2) \qquad (5)$

This F_u is the same as that of the non-symmetrical mechanism given in page 171.

3.2

In this calculation, the half portion of the failure mechanism is considered because of its symmetry .

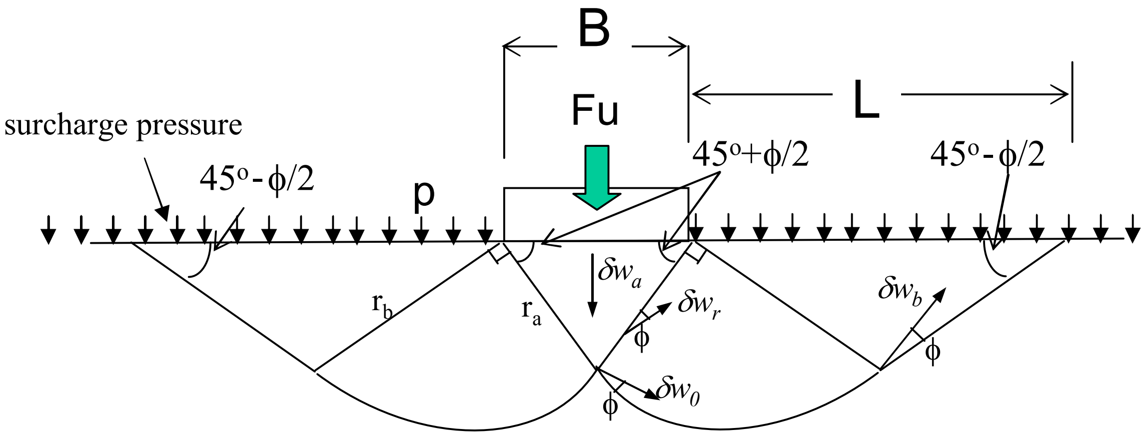

Fig.2.1

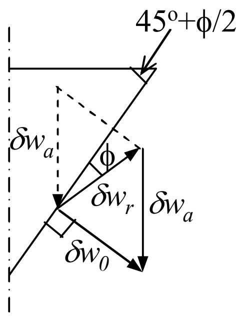

Fig.2.2

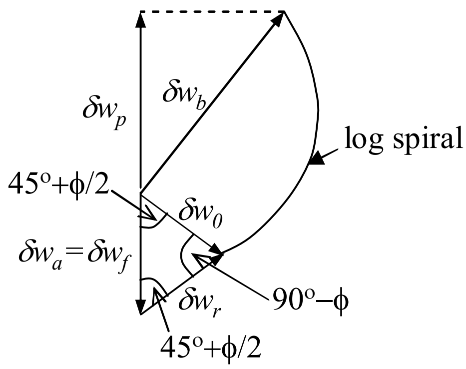

Fig.2.3

(1) From Fig. 2.1 and 2.2, displacement diagram can be drawn as shown in Fig.2.3.

(2) From the geometry shown in Fig.2.3,

$$\delta w_0 = \frac{\delta w_f}{2\cos(45^\circ + \phi/2)} \quad (1) \quad \text{and} \quad \delta w_b = \delta w_0 \exp(\frac{\pi}{2}\tan\phi) \quad (2)$$

from (1) and (2)

$$\delta w_p = \frac{\delta w_f}{2}\frac{\cos(\pi/4-\phi/2)}{\cos(\pi/4+\phi/2)}\exp(\frac{\pi}{2}\tan\phi) = \frac{\delta w_f}{2}\tan(\pi/4+\phi/2)\exp(\frac{\pi}{2}\tan\phi) \quad (3)$$

and also from Fig.1,

$$r_b = r_a \exp(\frac{\pi}{2}\tan\phi) \quad \textit{and} \quad L = B\tan(45^\circ + \phi/2)\exp(\frac{\pi}{2}\tan\phi) \quad (4)$$

For dry weightless soil, u=γ=0,

$$\delta W = 0 \qquad (5)$$

$$\delta E / 2 = \frac{F_u}{2}\delta w_f - pL\delta w_p \qquad (6)$$

using eqs. (3) and (4)

$$\delta E = \underline{F_u \delta w_f - pB\delta w_f \tan^2(45^\circ + \phi/2)\exp(\pi \tan\phi) \quad (7)}$$

(3) equating δE=δW and making use of eqs. (5) and (7)

$$\underline{F_u = pB\tan^2(45^\circ + \phi/2)\exp(\pi\tan\phi) \quad (8)}$$

Again F_u for symmetrical mechanism is the same as that of the non-symmetrical mechanism given in page 217 for weightless and non-cohesive soil.